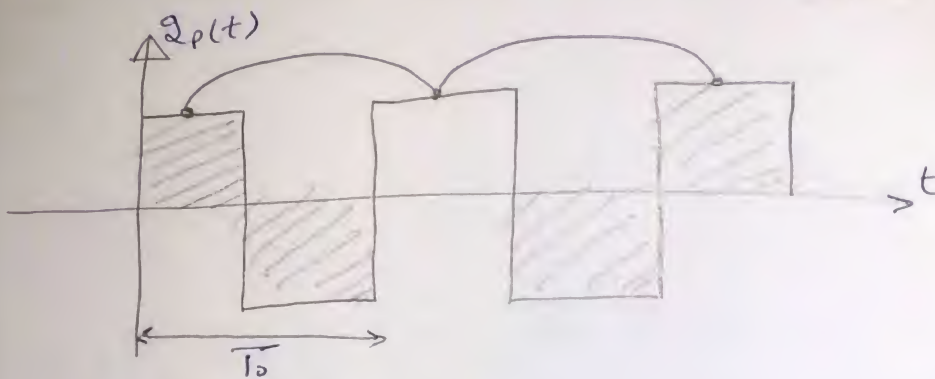


(حل) (1) المسألة

c.10/e.10

Fourier Series

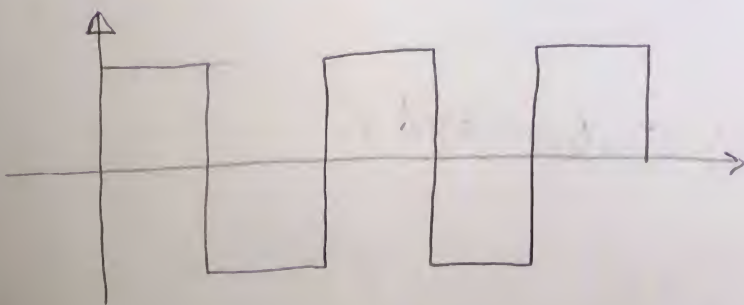
* F.s used to expand periodic signals into an infinite sum of series & Cosines.



→ For Periodic Signals

$$q_p(t) (t \pm mT_0) = q_p(t) \quad , m \text{ integer.}$$

→ For the previous wave :-



* There is

① DC $\rightarrow a_0$

② infinite sines

③ " C-sines

$$f_0 \rightarrow \text{Fundamental Freq.} = \frac{1}{T_0}$$

→ يوجد عدد لا نهائي من sines و cosines بالترتيب

$$\text{Sines} \rightarrow (f_0, 2f_0, \dots, \infty)$$

ف. 5/

$$g_p(t) = a_0$$

$$+ 2 \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t)$$

$$+ 2 \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

→ Trigonometric F.s

→ Coeff:-

$$* a_0 = \frac{1}{T_0} \int_0^{T_0} g_p(t) dt \quad \rightarrow \text{avg. of } g_p(t)$$

$$= \frac{\text{Area under } g_p(t) \text{ on interval of } T_0}{T_0}$$

$$* a_n = \frac{1}{T_0} \int_0^{T_0} g_p(t) \cos(n\omega_0 t) dt$$

$$* b_n = \frac{1}{T_0} \int_0^{T_0} g_p(t) \sin(n\omega_0 t) dt$$

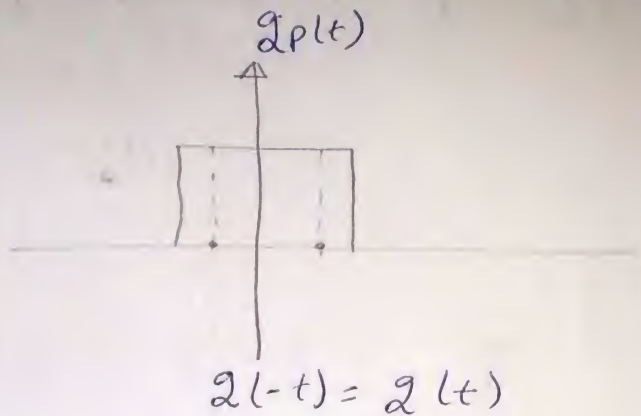
Notes

* if $q_p(t)$ is even f_n .

$$q_p(-t) = q_p(t)$$

$$\therefore b_n = 0$$

$$a_0, b_n \checkmark$$

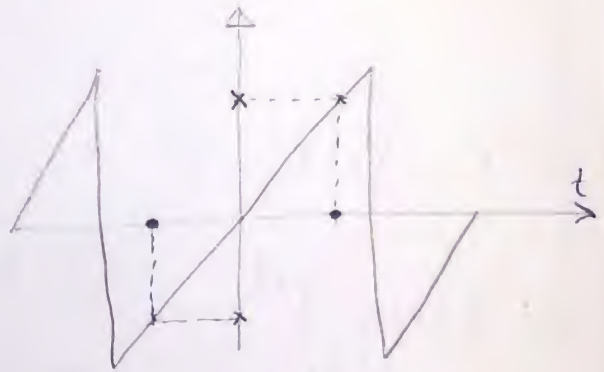


* if $q_p(t)$ is odd function

$$q_p(-t) = -q_p(t)$$

$$\therefore a_0 = a_n = 0$$

$$b_n \checkmark$$



خطوات حل أى مسألة

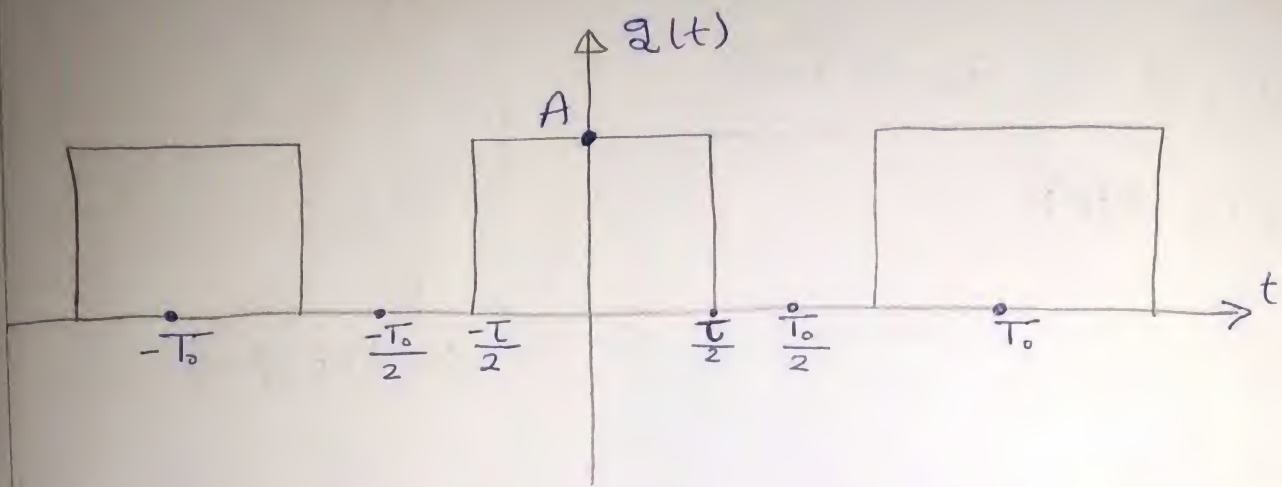
١- نحدد T_0 .

٢- نحدد معادلة الدالة خلال T_0 .

٣- "سؤال" هل الدالة زوجية أم فردية.

٤- نحسب a_0, a_n, b_n ونعوض في معادلة (Fourier series).

Ex: For the periodic Pulse train. Find F.s



$T \rightarrow$ Pulse width

$\frac{-T_0}{2} \rightarrow \frac{T_0}{2}$ ← نأخذ الفترة

① T_0 From $\frac{-T_0}{2}$ to $\frac{T_0}{2}$

$$\textcircled{2} \quad g(t) = \begin{cases} A & -\frac{T}{2} \leq t \leq \frac{T}{2} \\ 0 & \text{otherwise} \end{cases}$$

③ $g(t)$ is even fn. $\Rightarrow b_n = 0$

$$\begin{aligned} \textcircled{4} \quad a_0 &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g_p(t) dt \\ &= \frac{1}{T_0} \int_{-T/2}^{T/2} A \cdot dt \end{aligned}$$

ϵ

$$a_0 = \frac{A}{T_0} \left| t \right|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{A\tau}{T_0}$$

$$a_0 = \frac{A\tau}{T_0}$$

$$a_n = \frac{1}{T_0} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} p(t) \cos(n\omega_0 t) dt$$

$$= \frac{1}{T_0} \int_{-\frac{\tau}{2}}^{\tau/2} A \cdot \cos(n\omega_0 t) dt$$

$$a_n = \frac{A}{n\omega_0 T_0} \left| \sin(n\omega_0 t) \right|_{-\frac{\tau}{2}}^{\frac{\tau}{2}}$$

$$= \frac{A}{n \cdot T_0 \cdot \frac{2\pi}{T_0}} \left[\sin\left(n\omega_0 \frac{\tau}{2}\right) - \sin\left(n\omega_0 \left(-\frac{\tau}{2}\right)\right) \right]$$

$$a_n = \frac{A}{2n\pi} \cdot 2 \sin\left(n\omega_0 \frac{\tau}{2}\right)$$

$$a_n = \frac{A}{n\pi} \sin\left(\frac{n\pi\tau}{T_0}\right)$$



$$\therefore g_p(t) = a_0 + 2 \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + 2 \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$g_p(t) = \frac{AT}{T_0} + 2 \sum_{n=1}^{\infty} \frac{A}{n\pi} \sin\left(\frac{n\pi T}{T_0}\right) * \cos(n\omega_0 t)$$

Complex F.s

بعد اجراء بعض الاختهات
في المعادلة السابقة اوتي
القانون السابق .

$$g_p(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{+jn\omega_0 t}$$

$$C_n = \frac{1}{T_0} \int_{T_0} g_p(t) \cdot e^{-jn\omega_0 t}$$

← هذه الطريقة تستخدم لرسم (Spectrum).

← الإشارة مع ال (Frequency) هي (Spectrum)

$$C_n = |C_n| \cdot e^{j\theta_n}$$

$$|C_n| = \sqrt{a_n^2 + b_n^2}$$

$$|C_0| = \sqrt{a_0^2 + \underbrace{b_0^2}_0} \Rightarrow \boxed{|C_0| = a_0}$$

Power in F.S

* For any periodic signal $g_p(t)$, the avg. Power content is given by:

$$P_{avg} = \frac{1}{T_0} \int_0^{T_0} g_p^2(t) dt$$

or

$$P_{avg.} = \sum_{n=1}^{\infty} |C_n|^2$$